Inertial Mass Energy Equivalence

J.P. Wesley

Abstract

Introducing an inertial mass equivalent of the Coulomb potential energy, $M = -U_0/c^2$, the rate at which U_0 decreases as a charge q recedes from a fixed charge q' equals the rate of increase in kinetic energy, $dU_0/dt = -\mathbf{V}\cdot d[(m - U_0/c^2)\mathbf{V}]/dt$, where m is the material mass of q. Integrating, the total energy is $E = c^2m(1 - (1 - V^2/c^2)^{1/2}) + U_0(1 - V^2/c^2)^{1/2} \approx U_0 + (m - U_0/c^2)V^2/2$. The portion $U = (qq'/R)(1 - V^2/2c^2)$ is the Weber velocity potential. The net mass of an electron, $m_e - eV/c^2$, in a uniform electrostatic potential field ζ has been measured as a function of ζ. Applying the Weber theory to gravitation, $-Gmm'$ replacing qq', the far masses in the universe yield the force $F = (m\Phi_0/c^2)a = -(U/c^2)a$ in agreement with Mach's principle and inertial mass–potential energy equivalence. Associating an inertial mass with the kinetic energy K yields neomechanics, where $K = c^2m(1/(1 - v^2/c^2)^{1/2} - 1)$.

Key words: inertial mass–energy equivalence, Weber potential, Mach's principle, neomechanics

1. DERIVATION OF THE WEBER POTENTIAL

The concept of mass-energy equivalence is generally limited to the idea that material mass can be converted to active energy, such as thermal or radiant energy. Here the concept is applied to the inertial mass M equivalent to the electrostatic potential energy U_0 such that

$$M = -\frac{U_0}{c^2}, \tag{1}$$

where the Coulomb potential energy is

$$U_0 = \frac{qq'}{R}, \tag{2}$$

where q is a charge at $\mathbf{r}$, q' is a charge at $\mathbf{r}'$, and $R = |\mathbf{r} - \mathbf{r}'|$.

The rate that q loses potential energy when receding from the fixed charge q' equals the rate that work is done on the charge q to increase its kinetic energy; thus,

$$\frac{dU_0}{dt} = -\mathbf{V}\cdot\frac{d(M^*\mathbf{V})}{dt} = -\mathbf{V}\cdot\frac{d[(m - U_0/c^2)]\mathbf{V}}{dt}, \tag{3}$$

where $\mathbf{V} = d\mathbf{R}/dt$, m is the material mass of the charge q, and M^*, the total mass, is

$$M^* = m - \frac{U_0}{c^2}. \tag{4}$$

Integrating (3) yields the total energy E as

$$E = c^2m + (U_0 - c^2m)\sqrt{1 - \frac{V^2}{c^2}}, \tag{5}$$

where the constant of integration has been chosen as $E - c^2m$. From this derivation of (5) the velocity V is the rate of separation of the charges, so

$$V = \frac{dR}{dt} = \frac{\mathbf{R}\cdot\mathbf{V}}{R}, \tag{6}$$

and it is not the general relative velocity $\mathbf{v} - \mathbf{v}' = d\mathbf{r}/dt - d\mathbf{r}'/dt$.

For small values of V^2/c^2 (5) yields

$$E = U_0 + \left(m - \frac{U_0}{c^2}\right)\frac{V^2}{2}. \tag{7}$$

The portion of this total energy given by

$$U = U_0\left(1 - \frac{V^2}{2c^2}\right) = \left(\frac{qq'}{R}\right)\left(1 - \frac{V^2}{2c^2}\right) \tag{8}$$

is the Weber[1] velocity potential. It may be seen that the Weber potential is a proper *potential* energy, since taking a time derivative of (7) yields $\mathbf{v} \cdot \mathbf{F} = \mathbf{v} \cdot m\mathbf{a} = -dU/dt$, where $\mathbf{F}$ is the force on q and $\mathbf{v}$ and $\mathbf{a}$ are the velocity and acceleration of q, and for U static $-dU/dt = -\mathbf{v} \cdot \nabla U$ in agreement with Newton's second law for a body of mass m accelerated in the potential field U.

2. THE WEBER THEORY IS RELATIVISTIC

The derivation of the Weber velocity potential (8) depends only upon the motion and energy of the charge q in the field of a fixed charge q'. However, if the two charges are assumed to exist in an otherwise empty universe, then R and $V = dR/dt$ in (8) are the relative separation distance and the relative velocity; so the Weber potential (8) may be regarded as a pure relativistic expression.

Since absolute space exists,[2] and the fundamental physical laws applicable in the laboratory depend upon this preferred absolute rest-frame, the relativistic nature of Weber's potential must mean that it can only be approximately valid. In particular, relativistic theories in general, such as classical celestial mechanics, are adequate approximations if the finite velocity of action c can be neglected. This means that time intervals of interest must be large so that $\Delta t > L/c$, where L is the size of the system of interest. Thus, action, proceeding with velocity c, is able to establish a steady-state equilibrium throughout the system of interest in the time Δt. Since the finite time of propagation of action is being neglected, effects may be assumed to occur instantaneously. The Weber theory is, thus, a valid approximation for slowly varying effects where time intervals of interest are large.

3. THE MIKHAILOV EXPERIMENT MEASURING THE INERTIAL MASS EQUIVALENT OF POTENTIAL ENERGY

The Mikhailov[3] experiment verifies directly the existence of an inertial mass equivalent to the electrostatic potential energy, (1). He examined electrons moving in a uniform electrostatic potential field, where the total net mass of an electron is given by (4). An oscillating neon glow lamp was placed inside a hollow conducting sphere charged to the potential ζ to yield the desired uniform potential energy $U_0 = e\zeta$. The frequency of the glow lamp depends upon the mass of the electron M^*, as given by (4). Varying the electrostatic potential on the conducting sphere from -3000 to +3000 V, he found a linear decrease in the ratio $(M^* - m_e)/m_e$, as expected from the theory, where, according to (4), this ratio is $-U_0/c^2m_e = -e\zeta/c^2m_e$. For 3000 V he found $(M^* - m_e)/m_e = -(3.0 \pm 0.3) \times 10^{-3}$. According to the theory this ratio should be $-U_0/c^2m_e = -e\zeta/c^2m_e = -6 \times 10^{-3}$. The fact that the experimental value is one half the theoretical is not significant here, as the order-of-magnitude agreement is satisfactory, and there seem to be possibilities for systematic errors.

An alternative independent method for measuring the mass of the electron in a uniform electrostatic field should be undertaken as a check on the theory and on Mikhailov's results.

4. WEBER GRAVITATION AND MACH'S PRINCIPLE

The relativity approximation is appropriate for gravitation, where only slowly varying effects are involved and $V^2/c^2 \ll 1$. Thus, replacing qq' by $-Gmm'$ in (8) yields

$$U = -\left(\frac{Gmm'}{R}\right)\left(1 - \frac{V^2}{2c^2}\right), \tag{9}$$

which may be regarded as the Weber potential applied to gravitation.[4-7]

The force on the mass m due to a static mass m' may be obtained from

$$\frac{dU}{dt} = -\mathbf{v} \cdot \mathbf{F}, \tag{10}$$

where U is given by (9). The Weber force is thus found to be

$$\mathbf{F} = -G\left(\frac{mm'\mathbf{R}}{R^3}\right)\left[1 + \frac{v^2}{c^2} - \frac{3(\mathbf{R} \cdot \mathbf{v}/cR)^2}{2} + \frac{\mathbf{R} \cdot \mathbf{a}}{c^2}\right], \tag{11}$$

where $\mathbf{a} = d\mathbf{v}/dt$ is the acceleration of the mass m.

For a continuous distribution of static mass, m' may be replaced by

$$m' \rightarrow d^3r'\rho'(r') \tag{12}$$

and (11) integrated over all space to yield

$$\frac{\mathbf{F}}{m} = \left(1 + \frac{\mathbf{v}}{2c^2}\right)\nabla\Phi - \frac{\mathbf{v}(\mathbf{v} \cdot \nabla)\Phi}{c^2} - \frac{\Phi\mathbf{a}}{c^2} + \frac{(\mathbf{v} \cdot \nabla)^2\mathbf{H}}{2c^2} + \frac{(\mathbf{a} \cdot \nabla)\mathbf{H}}{c^2}, \tag{13}$$

where the gravitational potentials are defined by

$$\Phi = G\int \frac{d^3r'\rho'}{R} \text{ and } \mathbf{H} = G\int \frac{d^3r'\rho'\mathbf{R}}{R}. \tag{14}$$

The correctness of (13) may be established by reversing the steps: by introducing (14) into (13), carrying out the indicated differential operations under the integral sign, and finally replacing $\int d^3r'\rho'$ by m'.

In an infinite uniform isotropic universe in-the-large (the cosmological principle) the potentials $\mathbf{\Phi}$ and $\mathbf{H}$ produced by the far masses in the universe cannot vary locally from point to point, so all of the terms involving differentiation with respect to V in (13) must vanish. Thus, the force produced by the far masses in the universe is given simply by $-m(\Phi_0/c^2)\mathbf{a}$. In order to accelerate a body a local force must act against this force produced by the far masses in the universe; thus,

$$\mathbf{F}(\text{local}) = \left(\frac{m\Phi_0}{c^2}\right)\mathbf{a} = -\left(\frac{U}{c^2}\right)\mathbf{a} = m\mathbf{a}, \tag{15}$$

which is seen to be Newton's second law, which then satisfies Mach's principle, as well as the equivalence of inertial mass and potential energy, provided Weber's cosmological condition[8]

$$\Phi_0 = c^2 \tag{16}$$

is satisfied.

It may be noted that mathematically this result corresponds to the inertial properties of an electric charge moving in a uniform electrostatic potential field, such as produced inside a large hollow "distant" charged conducting sphere, the situation for the Mikhailov experiment (Section 3 above).

5. DERIVATION OF NEOMECHANICS

Neomechanics is Newtonian mechanics in absolute space-time extended to include the inertial mass equivalent of the kinetic energy K of a body. From Newton's second law the time rate of increase of the kinetic energy is then given by

$$\frac{dK}{dt} = \frac{\mathbf{v}\cdot d[(m + K/c^2)\mathbf{v}]}{dt}, \tag{17}$$

where the total inertial mass M is given by

$$M = m + \frac{K}{c^2}, \tag{18}$$

and where m is the rest mass equal to M when $K = 0$.

Integrating (17) yields

$$K = c^2 m(\gamma - 1) = c^2 m\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right), \tag{19}$$

where the constant of integration has been chosen so that $K = 0$ when $\mathbf{v} = 0$.

The total inertial mass M, from (18) and (19), is then

$$M = m\gamma = \frac{m}{\sqrt{1 - v^2/c^2}}. \tag{20}$$

From inertial mass–energy equivalence this total inertial mass M, (20), implies a total energy E given by

$$E = c^2 m\gamma = K + c^2 m, \tag{21}$$

which equals the kinetic energy plus a rest-mass energy given by

$$E_0 = c^2 m. \tag{22}$$

Newton's second law for neomechanics becomes, using (20),

$$\mathbf{F} = \frac{d(m\gamma\mathbf{v})}{dt}. \tag{23}$$

The problem in neomechanics, as in Newtonian mechanics, involves integrating Newton's second law, but as given by (23) instead of by $\mathbf{F} = m\mathbf{a}$.

The empirical validity of mass change with velocity, (20), or neomechanics, was first indicated by Kaufmann's[9] experiments. But due to the experimental uncertainties and the uncertainty in the Maxwell electromagnetic theory that was assumed, his results were not conclusive. However, Bertozzi,[10] by relating the time-of-flight velocity of electrons v to their kinetic energy K, known from the accelerating electric potential difference, was able to confirm (19) as reasonable, even for electron velocities approaching the velocity c. He was thus able to confirm mass change with velocity, (20).

Since neomechanics involves absolute space-time, the Monstein–Wesley[11] experiment, measuring the absolute velocity of the solar system from the anisotropy of the cosmic muon flux, is of considerable importance. It demonstrates for the first time empirically that the velocity v in the gamma factor $\gamma = 1/(1 - v^2/c^2)^{1/2}$ is the unique *absolute* velocity and not some arbitrary relative velocity. The Monstein–Wesley experiment helps to confirm empirically the validity of neomechanics in absolute space-time.

6. DISCUSSION

It is important to note the limitations of the present theory. It does not provide all of the needed answers.

The association of an inertial mass with potential energy yields a valuable extension of classical potentials not involving v^2/c^2 to velocity-dependent potentials involving v^2/c^2. But the extension is still limited to slowly varying effects, relativity, and $v^2/c^2 \ll 1$. It does not thus provide the electrodynamics needed for rapidly varying effects, for v^2/c^2 approaching unity, or for radiation. The search for a really adequate electrodynamics that includes the empirical successes of the Weber and Maxwell theories without their limitations and failures must continue.[12]

The association of inertial mass with kinetic energy, yielding the extension of the inertial force from $m\mathbf{a}$ to $d(m\gamma\mathbf{v})/dt$, is not limited to slowly varying effects, to relativity, or to $v^2/c^2 \ll 1$. However, the theory has still not been adequately empirically confirmed *quantitatively*. The kinetic energy of a particle, such as an electron, needs to be measured *accurately* as a function of its time-of-flight velocity, a functional relationship that is independent of any questionable electrodynamics. This all-important experiment should be easily performed using one of the large particle accelerators currently available.

The mass equivalent of the total internal energy of a closed system has been used to derive the Bethe–Weizsäcker mass formula for the masses of the elements using a Lennard–Jones nucleon-nucleon potential.[13] In this case a mass equivalent is identified with plus the potential energy, $M' = +U/c^2$, which would seem to conflict with the negative sign used in (1) above. The situations are, however, quite different. The *inertial* mass equivalent in (1) is a small *second-order correction*. For example, in (7) the total energy involves the usual kinetic and potential energies minus the correction as a small decrease in the kinetic energy $-(U_0/c^2)V^2/2$. The mass equivalent of the total energy $M' = E/c^2$ would thus include this small negative second-order correction as $-[U_0/c^2)V^2/2]/c^2$, which varies as $1/c^4$.

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Résumé

Pour représenter une masse inertielle équivalente à l'énergie de Coulomb, $M = -U_0/c^2$, la vitesse avec laquelle U_0 diminue, quand une charge q recule d'une charge q' fixe, est égale au taux d'augmentation de l'énergie cinétique, $dU_0/dt = -V \cdot d[(m - U_0/c^2)V]/dt$, où m est la masse matérielle de q. Intégré, l'énergie totale est $E = c^2m(1 - (1 - V^2/c^2)^{1/2}) + U_0(1 - V^2/c^2)^{1/2} \approx U_0 + (m - U_0/C^2)V^2/2$. La portion $U = (qq'/R)(1 - V^2/2c^2)$ est le potentiel dépendant de la vitesse de Weber. La masse nette d'un électron, $m_e - eV/c^2$, dans un champ de potentiel uniforme ζ, a été mesuré en fonction de ζ. Pour appliquer la théorie de Weber à la gravitation, $-Gmm'$ remplaçant qq', les masses éloignées dans l'univers rendent $F = (m\Phi_0/c^2)a = -(U/c^2)a$ s'accorde avec le principe de Mach et la masse inertielle équivalente à l'énergie potentielle. Pour associer une masse inertielle avec l'énergie cinétique K, donne une néomécanique, où $K = c^2m(1/(1 - v^2/c^2)^{1/2} - 1)$.

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J.P. Wesley
Weiherdammstrasse 24
78176 Blumberg
Germany