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# Einstein Dynamics Without Special-Relativistic Kinematics

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*The Michelson–Morley result is described empirically by generalized Doppler equations. If the phase of a light wave is not invariant, in agreement with the quantum nature of light, special-relativistic kinematics need not be assumed. Einstein particle dynamics and Maxwell–Lorentz electrodynamics in a moving system are derived without assuming special-relativistic kinematics. An alternative explanation for the decay rate of moving radioactive particles is presented. The observation of a third-order Doppler effect may yield the velocity of the closed laboratory.*

## 1. INTRODUCTION

The Lorentz symmetry of special relativity[1] provides a powerful analytical tool for modern theoretical physics. Yet it seems to lead to inconsistencies and paradoxes when applied to spacetime.[2–8] Moreover, observations and experiments have been interpreted as supporting "absolute" rather then "relativistic" spacetime.[9–13] It therefore becomes of interest to explore the possibility of preserving Lorentz symmetry for dynamical quantities, such as momentum, energy, and force, i.e., Einstein dynamics, while leaving the choice of kinematics open.

## 2. EMPIRICAL DESCRIPTION OF THE MICHELSON–MORLEY RESULT

The Michelson–Morley result[14–18] shows that a standing wave pattern for light remains invariant to its orientation on the moving earth. The phase

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velocity (but not necessarily the oneway chopped time-of-flight velocity) of light must be determined to fit this result. From the definition of phase velocity $c$ the function $c^2k^2 - \omega^2$, where $\mathbf{k}$ is the propagation constant and $\omega$ the angular frequency, is an invariant. In particular,

$$c'^2k'^2 - \omega'^2 = c^2k^2 - \omega^2 \tag{1}$$

where unprimed quantities refer to a massive system at rest and primed quantities refer to the massive system when moving with the velocity $\mathbf{v}$. By linearizing Eq. (1) symmetrically and by assuming that components of $c\mathbf{k}$ transverse to $\mathbf{v}$ remain unchanged, we obtain a Lorentz symmetric comparison

$$\begin{aligned} c'k_x' &= c\gamma(k_x - \omega v/c^2), \qquad & c'k_y' &= ck_y \\ c'k_z' &= ck_z\,, & \omega' &= \gamma(\omega - k_x v) \end{aligned} \tag{2}$$

where

$$\gamma = (1 - v^2/c^2)^{-1/2} \tag{3}$$

where the $x$ axis is in the direction of $\mathbf{v}$.

This result, Eqs. (2) and (3), implies a phase velocity given by

$$c_x' = f(v)(c_x - v), \qquad c_y' = f(v)c_y/\gamma \tag{4}$$

where $f(v)$ is any arbitrary function of the velocity $v$, since $c'$ and $k'$ cannot be simultaneously and independently specified. This result (4) may be readily shown to satisfy the Michelson–Morley result for any $f(v)$. If the phase velocity $c' = c$, as in special relativity, then $f(v) = (1 - c_xv/c^2)^{-1}$. If the phase velocity magnitude has the classical value $c' = c(1 - c_xv/c^2)$, then $f(v) = 1$. The discussion that follows is independent of any particular choice of $f(v)$ or $c'$.

Equations (2) may be interpreted as generalized Doppler equations which represent an empirical description of the Michelson–Morley result. They represent the Einstein–Doppler relations for the special case $c' = c$.

## 3. THE PHASE OF A LIGHT WAVE IS NOT INVARIANT

According to the classical wave theory of light the propagation constant $\mathbf{k}$ and the frequency $\omega$ are pure kinematical properties and the phase of a wave $\mathbf{k} \cdot \mathbf{r} - \omega t$ is an invariant. According to the quantum mechanical nature of light the propagation constant $\mathbf{k}$ and the frequency $\omega$ are essentially dynamic properties given by the de Broglie wavelength and the Panck–Einstein frequency condition; thus,

$$\mathbf{k} = \mathbf{p}/\hbar, \qquad \omega = E/\hbar \tag{5}$$

where $h = 2\pi\hbar$ is Planck's constant and $\mathbf{p}$ is the momentum and $E$ the energy of a photon. In a moving system Eqs. (5) are generalized to

$$c'\mathbf{k}'/c = \mathbf{p}'/\hbar, \qquad \omega' = E'/\hbar \tag{6}$$

The $\mathbf{k}$ and $\omega$ that will be observed in general must be determined from quantum theory and dynamical considerations. In general $\mathbf{k}$ and $\omega$ cannot be derived using classical wave theory and pure kinematical considerations. For example, a large molecule will see absorbed light as having a higher frequency than a small molecule, the recoil energy of the large molecule being less than the recoil energy of the small molecule [Eq. (18) below]. Classically the frequency would have to remain the same independent of the mass of the molecule. The quantum mechanical method of determining $\mathbf{k}$ and $\omega$ dynamically does not generally preserve the phase of a light wave. The classical invariance of the phase may therefore be abandoned; or,

$$(c'\mathbf{k}'/c) \cdot \mathbf{r}' - \omega' t' \neq \mathbf{k} \cdot \mathbf{r} - \omega t \tag{7}$$

If the classical invariance of the phase were to be assumed, then Eqs. (2) for $c' = c$, when interpreted as a passive transformation of the point of view, would lead to special-relativistic kinematics. But when Eq. (7) is assumed, Galilean kinematics, or other possible choices for the kinematics, may be considered.

## 4. INTERPRETATION OF THE GENERALIZED DOPPLER EQUATIONS

Since Eqs. (2) do not represent the classical Doppler effect and if special-relativistic kinematics is not assumed, then Eqs. (2) must represent in part a dynamical interaction of the observer with the wave being observed. The state of motion of the massive observer alters the wave physically, which then helps to determine what is observed. The active role of the observer in the measuring process is already familiar in quantum theory; and light is a quantum mechanical phenomenon.

With this dynamical interpretation the generalized Doppler equations (2) do not represent a transformation between the points of view of two passive observers of a single physical process or event. Instead, Eqs. (2) compare two different physical processes or events, one where a stationary observer actively affects that which is observed, and the other where a moving observer actively affects differently that which is observed.

This physical interpretation may also be applied to other dynamical quantities exhibiting the same Lorentz symmetric comparisons as Eqs. (2).

## 5. EINSTEIN DYNAMICS FOR A PARTICLE OF A FINITE MASS

Substituting Eqs. (5) and (6) into (2) yields the comparison

$$\begin{aligned} p_x' &= \gamma(p_x - Ev/c^2), \qquad & p_y' &= p_y \\ p_z' &= p_z\,, & E' &= \gamma(E - p_x v) \end{aligned} \tag{8}$$

where primes denote quantities observed in the massive, moving system. It is now postulated that Eqs. (8) are also valid for particles of a finite mass. As discussed above, these Eqs. (8) do not represent a transformation.

To obtain the dynamics that Eqs. (8) imply, the special case when the momentum of the particle is zero in the massive, moving system, $\mathbf{p}' = 0$, may be considered. In this case Eqs. (8) reduce to

$$p_x = vE/c^2, \qquad p_y = p_z = 0, \qquad E' = \gamma(E - p_x v) \tag{9}$$

where $E'$ cannot be zero. Imposing the additional condition that the momentum reduce to the Newtonian expression for small velocities, we obtain from Eqs. (9)

$$E' = mc^2, \qquad \mathbf{p} = m\gamma\mathbf{v}, \qquad E = m\gamma c^2 \tag{10}$$

where $m$ is the mass of the particle and $\gamma$ is given by Eq. (3). Equations (10) are the usual equations for Einstein dynamics for a particle of finite mass. This derivation does not require special-relativistic kinematics.

## 6. ELECTRODYNAMICS IN A MOVING SYSTEM

Since a light wave is seen in a massive, moving system as well as in a massive, stationary system, the wave equation, and consequently Maxwell's equations, may be assumed to be of the same form in a massive, moving system as in a massive, stationary system. The homogeneous Maxwell equations in Gaussian form[19] for a plane wave solution may be written in the form

$$c\mathbf{k} \times \mathbf{E} - \omega\mathbf{B} = 0, \qquad c\mathbf{k} \cdot \mathbf{B} = 0$$
$$c\mathbf{k} \times \mathbf{H} + \omega\mathbf{D} = 0, \qquad c\mathbf{k} \cdot \mathbf{D} = 0 \tag{11}$$

Substituting Eqs. (2) into (11) and preserving the form of Maxwell's equations yields

$$\begin{aligned} &\mathbf{E}_\parallel' = \mathbf{E}_\parallel, \quad \mathbf{B}_\parallel' = \mathbf{B}_\parallel, \quad \mathbf{D}_\parallel' = \mathbf{D}_\parallel, \quad \mathbf{H}_\parallel' = \mathbf{H}_\parallel \\ &\mathbf{E}_\perp' = \gamma(\mathbf{E}_\perp + \mathbf{v} \times \mathbf{B}/c), \qquad \mathbf{B}_\perp' = \gamma(\mathbf{B}_\perp - \mathbf{v} \times \mathbf{E}/c) \\ &\mathbf{D}_\perp' = \gamma(\mathbf{D}_\perp + \mathbf{v} \times \mathbf{H}/c), \qquad \mathbf{H}_\perp' = \gamma(\mathbf{H}_\perp - \mathbf{v} \times \mathbf{D}/c) \end{aligned} \tag{12}$$

where primes refer to fields observed in the moving system, and the subscript $\parallel$ refers to components parallel to $\mathbf{v}$ and $\perp$ to components perpendicular to $\mathbf{v}$. Since Eqs. (12) do not involve either $c\mathbf{k}$ or $\omega$, they may be regarded as generally valid comparisons for all solutions to Maxwell's equations.

Introducing sources into Maxwell's equations and using Eqs. (12) yields by symmetry

$$\begin{aligned} &J_x' = \gamma(J_x - \rho v), \qquad J_y' = J_y \\ &J_z' = J_z, \qquad \rho' = \gamma(\rho - J_x v/c^2) \end{aligned} \tag{13}$$

where $\mathbf{J}$ and $\rho$ are the electric current and charge densities. The force in the massive, moving system is then the Lorentz force,

$$\mathbf{F}' = \rho'\mathbf{E}' + \mathbf{J}' \times \mathbf{B}'/c \tag{14}$$

These results, Eqs. (12)–(14), do not represent a transformation of the passive point of view, as discussed in Section 4. Special-relativistic kinematics was not used for the derivation of these results.

## 7. THE HALF-LIFE OF MOVING RADIOACTIVE PARTICLES

The half-life $\tau'$ of moving radioactive particles is increased according to the formula[20–22]

$$\tau' = \gamma\tau \tag{15}$$

where $\tau$ is the half-life for stationary particles and $\gamma$ is given by Eq. (3). This result is frequently interpreted as a special-relativistic time dilation. Conventional relativistic quantum theory provides another interpretation.[23]

The probability density for a free particle is chosen proportional to $mc^2/E = 1/\gamma$, which means that the probability of decay is also proportional to $1/\gamma$, in agreement with Eq. (15). The normalizing factor $mc^2/E = 1/\gamma$ arises due to the Lorentz contraction of the proper element of volume in the normalizing integral. Conventional relativistic quantum theory thus explains the effect as essentially a Lorentz contraction. The space interval in which the particle can be found decreases with its velocity as $1/\gamma$.

The present theory, seeking alternatives to special-relativistic kinematics, does not accept these interpretations. If Galilean kinematics is assumed, the following interpretation is possible: The probability of decay is proportional to the statistical weight of the final state, which is proportional to the volume of phase space available for the free daughter particles. The volume element of momentum space in the center-of-mass system as compared with that in the stationary system may be obtained by taking differentials of Eqs. (8) for $E'$ constant, which yields

$$dp_x'\, dp_y'\, dp_z' = dp_x dp_y\, dp_z/\gamma \tag{16}$$

Using Galilean kinematics, the element of volume in configuration space is found to remain the same, $dx'\, dy'\, dz' = dx\, dy\, dz$; so that the volume of phase space which is available for the daughter particles is statistically less by the factor $1/\gamma$; and the result (15) follows.

## 8. THE GENERAL DOPPLER EFFECT

The Doppler effect for a moving source of mass $M_s$ and velocity $\mathbf{v}_s$ and a massive, stationary observer may be obtained by considering the emission of a single photon of angular frequency $\omega$. Considering conservation of momentum and energy and using Eqs. (5) and (10), we obtain the result

$$\omega = \omega_0(1 - \hbar\omega_0/2M_s c^2)/\gamma_s(1 - \mathbf{v}_s \cdot \mathbf{c}/c^2) \tag{17}$$

where $\omega_0$ is the frequency for an infinitely massive stationary source and $\gamma_s$ is given by Eq. (3) with $v_s$ replacing $v$. The Doppler effect for a moving observer of mass $M_0$ and velocity $\mathbf{v}_0$ and a massive stationary source may be similarly obtained by considering the absorption of a single photon of frequency $\omega$, yielding

$$\omega'/K = -1 + [1 + 2\omega\gamma_0(1 - \mathbf{v}_0 \cdot \mathbf{c}/c^2)/K]^{1/2} \tag{18}$$

where $\omega'$ is the frequency of the absorbed light, $\gamma_0$ is given by Eq. (3) with $v_0$ replacing $v$, and $K = M_0 c^2/\hbar$.

The general Doppler effect when both source and observer are in motion may be obtained by substituting Eq. (17) for $\omega$ into (18). The case when both the source and observer are massive may then be obtained by letting $M_s \to \infty$ and $M_0 \to \infty$, yielding

$$\omega' = \omega_0 \gamma_0 (1 - \mathbf{v}_0 \cdot \mathbf{c}/c^2)/\gamma_s(1 - \mathbf{v}_s \cdot \mathbf{c}/c^2) \tag{19}$$

These results, Eqs. (17)–(19), have been derived without having to assume special-relativistic kinematics. It is therefore possible to consider Galilean kinematics and the possible influence of the velocity of the closed laboratory on the general Doppler effect. The laboratory velocity **V** may be introduced by letting

$$\mathbf{v}_s = \mathbf{V} + \mathbf{u}_s, \qquad \mathbf{v}_0 = \mathbf{V} + \mathbf{u}_0 \tag{20}$$

where $\mathbf{u}_s$ and $\mathbf{u}_0$ are velocities of the source and observer relative to the laboratory. Laboratory coordinates may be chosen so that the $xy$ plane is defined by **V** and **c** (the phase velocity of light in the zero-velocity frame). The $y$ axis is taken in the direction of the phase velocity in the laboratory. Introducing Eqs. (20) into (19) and expanding to third powers in (velocity/$c$) yields

$$\omega'/\omega_0 = 1 + A_1 + A_2 + A_3 + O\{(\text{velocity}/c)^4\} \tag{21}$$

where the $A$'s are defined by

$$\begin{aligned}
cA_1 &= u_{sy} - u_{oy}, \qquad 2c^2 A_2 = (u_{sy} - u_{oy})^2 + u_{ox}^2 - u_{sx}^2 \\
2c^3 A_3 &= (u_{sy} - u_{oy})(u_{sy}^2 - u_{sx}^2 + u_o^2) + 2V_x(u_{sx}u_{sy} - u_{ox}u_{oy}) \\
&\quad - 2V_y(u_{sy} - u_{oy})^2 + V_x^2(u_{sy} - u_{oy}) \\
&\quad + 2V_xV_y(u_{sx} - u_{ox}) + 2V_y^2(u_{sy} - u_{oy}).
\end{aligned} \tag{22}$$

The laboratory velocity **V** does not appear in the first- and second-order terms in Eqs. (21) and (22), and these terms are sufficient to predict the various Doppler-effect experiments that have been performed to date.

In an attempt to observe the velocity of the closed laboratory, Champeney and Moon[24,25] rotated a rod with a Mössbauer source on one end and an absorber on the other end and kept a record of the gamma-ray counts as a function of the time of day and year. No variations were detected. This agrees with the present theory, where no variations should be expected to second order; although with greater care the third order term varying as $2V_x V_y(u_{sx} - u_{0x})$ might be detectable.

## 9. PROPOSAL TO DETECT THE CLOSED LABORATORY VELOCITY

The velocity of the closed laboratory **V** may be detected by going to an appropriate third-order Doppler effect. If a Mössbauer source is placed at the center of a rotating rod and an absorber at the end, Eqs. (21) and (22) yield to third order

$$\omega'/\omega_0 = 1 + u_{0x}^2/2c^2 - u_{0x}V^2 \cos^2\phi \sin(2\Omega t)/2c^3 \tag{23}$$

where $\Omega$ is the angular velocity of the rod, $\phi$ is the angle between the plane of rotation of the rod and **V**, and $u_{0x}$ is the tangential velocity of rotation of the absorber. The term involving $V$ may be singled out by amplifying the audio frequency $2\Omega$. Mechanical vibrations of this frequency may also arise; but the desired effect varies through a maximum twice daily, since $\phi = \phi(t)$, and ordinary vibrations should not be subject to such a regular daily variation. If $V$ is about 300 km/sec, the amplitude of the effect might be made to be of the order of $u_{0x}V^2/2c^3 \approx 10^{-13}$, which should be detectable.

A cesium beam clock, where the light is viewed normal to the motion of the atoms, is in principle subject to a daily fractional time variation given by the last term in Eq. (22), or by

$$\Delta\omega/\omega_0 = V_x V_y u_{sx}/c^3 \tag{24}$$

If the velocity $u_{sx}$ of the atoms is about 300 m/sec then the magnitude of the effect, Eq. (24), is again about $10^{-13}$ for a laboratory velocity of about 300 km/sec. If a cesium beam clock can be made to run with a fractional error of this order of magnitude, the effect might be just observable. Unfortunately, a rigidly mounted clock would be subject to extraneous daily effects which would probably mask the effect sought.

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