Displacement current

— and how to get rid of it

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To enable the continuity of electric current to be retained across a capacitor Maxwell proposed a "displacement current". By treating the capacitor as a special kind of transmission line this mathematical convenience is no longer required.

CONVENTIONAL electromagnetic theory proposes that when an electric current flows down a wire into a capacitor it spreads out across the plate, producing an electric charge which in turn leads to an electric field between the capacitor plates. The valuable concept of continuity of electric current is then retained by postulating (after Maxwell)[1] a "displacement current", which is a mathematical manipulation of the electric field E between the capacitor plates which has the dimensions of electric current and completes the flow of "electricity" (Fig. 1 (a) and (b)). This approach permits us to retain Kirchhoff's Laws and other valuable concepts, even though superficially it appears that at the capacitor there is a break in the otherwise continuous flow of electric current.

The flaw in this model is revealed when we notice that the electric current entered the capacitor at one point only on the capacitor plate. We must then explain how the electric charge flowing down the wire suddenly distributes itself uniformly across the whole capacitor plate. We know that this cannot happen since charge cannot flow out across the plate at a velocity in excess of the velocity of light. This paradoxical situation is brought about by a fundamental flaw in the basic model. Work on high speed logic design[2] has shown that the model of a lumped capacitance is faulty, and "displacement current" is an artefact of this faulty model.

The true model is quite different. Electric current enters the capacitor through a wire and then spreads out across the plate of the capacitor in the same way as ripples flow out from a stone dropped into a pond. If we consider only one pie-shaped wedge of the capacitor, as in Fig 1 (c), we can recognise it as a parallel plate transmission line whose only unusual feature is that the line width is increasing (and hence the impedance is decreasing). The capacitor is made up of a number of these pie-shaped transmission lines in parallel, so the proper model for a capacitor is a transmission line.

Equivalent series resistance for a capacitor is the initial characteristic impedance of this transmission line at a radius equal to the radius of the input wires. Series inductance does not exist. *Pace* the many documented values for series inductance in a capacitor, this confirms experience that when the so-called series inductance of a capacitor is measured it turns out to be no more than the series inductance of the wires connected to the capacitor. No mechanism has ever been proposed for an internal series inductance in a capacitor.

Since any capacitor has now become a transmission line, it is no more necessary to postulate "displacement current" in a capacitor than it is necessary to do so for a transmission line. The excision of "displacement current" from Electromagnetic Theory has been based on arguments which are independent of the classic dispute over whether the electric current causes the electromagnetic field or vice versa.

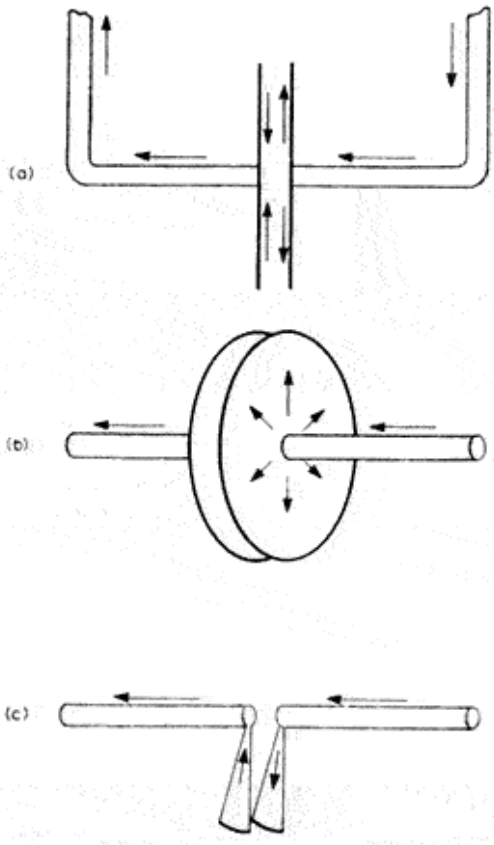


Fig. 1 *Process of current flowing into a capacitor and spreading out across a plate is shown in (a) and (b). The structure in (b) can be considered as being made up of a number of pie-shaped wedges as in (c), each of which is a transmission line.*

Appendix

Comparison of the transmission line model with the lumped model of a capacitor in an RC circuit.

Taking the above discussion further, consider a transmission line as shown in Fig. 2, assumed to be terminated with a resistance R_T (not shown). The reflectian coefficient is $\rho=(R_T-Z_o)/(R_T+Z_o)$ where Z_o is the characteristic impedance of the line. If the line is open-circuit at the right-hand end, as shown (and therefore R_T is infinite), the $\rho=+1$. We will assume that $R \gg Z_o$.

When switch S is closed (at time $t=0$) a step of voltage $V.Z_0/(R+Z_o)$ is propagated down the line. This reflects from the open circuit at the right hand end to give a total voltage $2V.Z_o/(R+Z_o)$. Reflection from the left end makes a further contribution of $[V.Z_0/(R+Z_0)] \times [(R-Z_0)/(R+Z_0)]$ and so on. In general after n two-way passes the voltage after n passes is V_n and,

$$V_{n+1}=V_n+2.\frac{VZ_0}{R+Z_0}\left[\frac{R-Z_0}{R+Z_0}\right]^n \qquad (1)$$

In order to avoid a rather difficult integration it is possible to sum this series to n terms using the formula,

$$=\frac{a(1-v^n)}{1-v} \qquad (2)$$

where a is the first term of a geometrical progression and v the ratio between terms. (This formula is easily verified by induction.) Substituting in (2) the parameters from (1),

$$\text{i.e. } a=\frac{2VZ_0}{R+Z_0} \qquad (3)$$

$$v=\frac{R-Z_0}{R+Z_0} \qquad (4)$$

We obtain,

$$V_n = \frac{\frac{2VZ_0}{R+Z_0}\left[1-\left[\frac{R-Z_0}{R+Z_0}\right]^n\right]}{1-\frac{R-Z_0}{R+Z_0}} \quad (5)$$

$$= V\left[1-\left[\frac{R-Z_0}{R+Z_0}\right]^n\right] \quad (6)$$

This is the correct description of what is happening as a capacitor charges. We can now go on to show that it is approximated by an exponential. We have

$$V_n = V\left[1-\left[\frac{R-Z_0}{R+Z_0}\right]^n\right] \quad (7)$$

Consider the term,

$$T = \left(\frac{R-Z_0}{R+Z_0}\right)^n$$

$$= \left(\frac{1-Z_0/R}{1+Z_0/R}\right)^n$$

If $Z_0/R \ll 1$ this term is asymptotically equal to

$$\left(1-\frac{2Z_0}{R}\right)^n$$

Now define $k = 2Z_0 n/R$. Substitution gives:

$$T = \left[1-\frac{k}{n}\right]^n$$

By definition, as $n \to \infty$ we have,

$$T = e^{-k} = e^{\frac{-2Z_0 n}{R}}$$

And therefore:

$$V_n = V\left[1-e^{\frac{-2Z_0 n}{R}}\right]$$

Now, after time t, $n = V_c t/2l$, where V_c = velocity of propagation.

Therefore

$$V(t) = V\left[1-e^{\frac{-V_c t}{l}\cdot\frac{Z_0}{R}}\right]$$

For any transmission line it can be shown that:

$$Z_0 = f\sqrt{\frac{\mu}{\epsilon}}$$

$$V_c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$C_1 = \epsilon/f$$

where C_1 = capacitance per unit length, and f is the same geometrical factor in each case. The "total capacitance" of length l of line $= lC_1 = C$.

$$\text{Hence } \frac{V_c Z_0}{lR} = \frac{1}{RC}$$

and therefore

$$V(t) = V(1-e^{-t/RC})$$

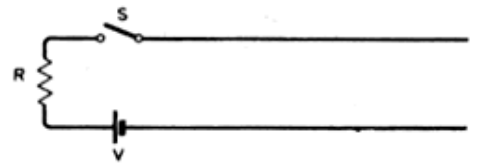
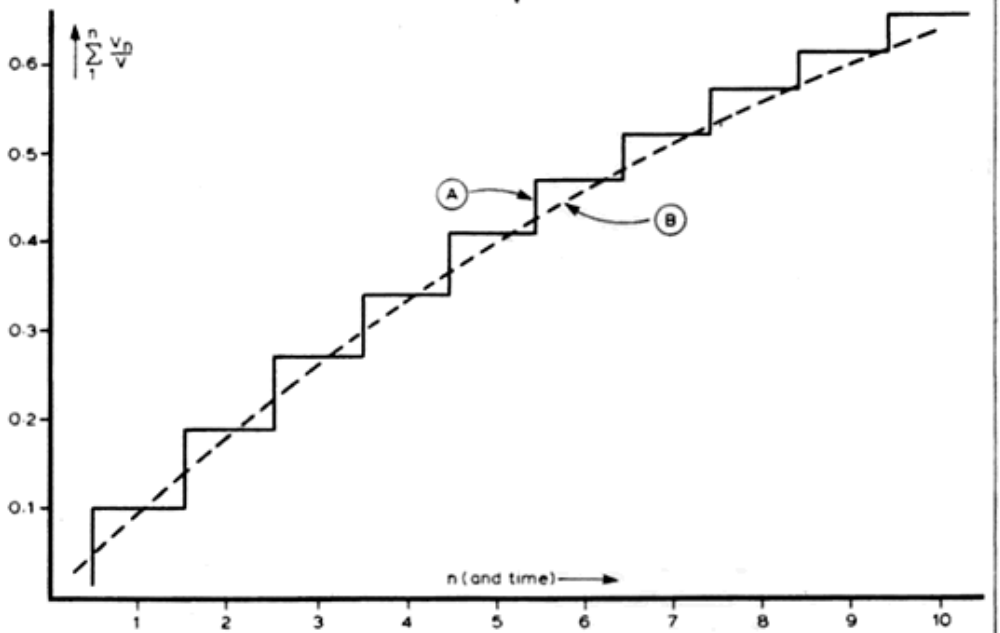


Fig. 2 *An open-ended transmission line.*

Fig. 3 *Comparison of the transmission line model $1-(1-2Z_0/R)^n$ in the curve A with the lumped model $1-e^{-2Z_0 n/R}$ in curve B, for $2Z_0/R = 0.1$.*



which is the standard result. This model does not require use of the concept of charge. A graphical comparison of the results is shown in Fig. 3.

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The death of electric current

A contribution to electromagnetic theory

by Ivor Catt CAM Consultants

Conventionally a signal can be understood either in terms of electricity in conductors, with associated fields, or in terms of electric and magnetic fields terminating on those conductors. In this article the author steps outside the accepted dualism and proposes a mechanism of signal transmission based on Oliver Heaviside's 'energy current' without recourse to 'conductors' in their conventional role.

A major advance in electromagnetic theory, which I shall call the transition from Theory N to Theory H, was made by Oliver Heaviside a century ago. What is proposed here is a transition from Theory H to a third theory, Theory C. It is to be hoped that the response to Theory C will be more perceptive than was the general response to Theory H a century ago, as typified by Sprague, quoted in this article. Until it was revived recently by CAM Consultants, Theory H had been ignored and then suppressed for a century. It was revived because of its great value in digital electronic design.[1,2]

Theory C has major implications across a whole spectrum of subjects. It could trigger an exciting renaissance in many fields of endeavour.

Whereas the conventional approach to electromagnetic theory is to concentrate on the electric current in wires, with some additional consideration of voltages between wires, Heaviside concentrates primarily on what he calls 'energy current', this being the electromagnetic field which travels in the dielectric between the wires. It has an amplitude equal to the Poynting Vector, $E \times H$. Heaviside's phrase, "We reverse this"; points to the great watershed in the history of electromagnetic theory – between the 'ethereals', who with Heaviside believe that the signal is an 'energy current' which travels in the dielectric between the wires, and the 'practical electricians', who like Sprague believe that the signal is an electric current which travels down copper wires, and that if there *is* a 'field' in the space between the wires, this is only a result of what is happening in the conductors.

Oliver Heaviside announced Theory H a century ago[3]:

"Now in Maxwell's theory there is the potential energy of the displacement produced in the dielectric parts by the electric force, and there is the kinetic or magnetic energy of the magnetic induction due to the magnetic force in all parts of the field, including the conducting parts. They are supposed to be set up by the current in the wire. We reverse this; the current in the wire is set up by the energy transmitted through the medium around it . . ."

The importance of Heaviside's phrase, "We reverse this;" cannot be overstated. It points to the watershed between the 'practical electricians', who have held sway for the last half century; promulgating their theory – which we shall call 'Theory N', the Normal Theory: that the cause is electric currents in wires and electromagnetic fields are merely an effect – and the 'ethereals', who believe what we shall call 'Theory H': that the travelling field is the cause, and electric currents are merely an effect of these fields.

Opposition to any attempted change from the familiar Theory N to Theory H was forceful and successful for the next century. Sprague, a 'practical electrician' wedded to Theory N, with its retention of a phlogiston-like 'fluid'*, electricity, at the centre of the electromagnetic stage, wrote[4]:

"A new doctrine is becoming fashionable of late years, devised chiefly in order to bring the now important phenomena of alternating currents under the mathematical system. It is purely imaginery . . . based upon Clerk-Maxwell's electromagnetic theory of light, itself described by a favourable reviewer as 'a daring stroke of scientific speculation,' alleged to be proved by the very little understood experiments of Hertz, and supported by a host of assumptions and assertions for which no kind of evidence is offered; but its advocates now call it the 'orthodox' theory.

"This theory separates the two factors of electricity . . ., and declares that the 'current', the material action, is carried by the 'so-called conductor' (which according to Dr Lodge contains nothing, not even an impulse, and according to Mr O. Heaviside is to be regarded as an obstructor), but the energy leaves the 'source' (battery or dynamo) 'radiant in exactly the same sense as light is radiant', according to Professor Silvanus P. Thompson, and is carried in space by the ether: that it then 'swirls' round (cause for such swirling no one explains) and finds its way to the conductor in which it then produces the current which is apparently merely an agency for clearing the ether of energy which tends to 'choke' it, while the conductor serves no other purpose than that of a 'waste pipe' to get rid of this energy . . .

"This much, however, is certain; that if the 'ether' or medium, or di-electrics carry the energy, the practical electrician must not imagine he can get nature to do his work for him; the ether, &c., play no part whatever in the calculations he has to make; whether copper wire is a conductor or a waste pipe, that is what he has to provide in quantity and quality to do the work; if gutta percha, &c., really carry the energy, he need not trouble about providing *for that purpose;* he must see to it that he provides it according to the belief that it prevents loss of current. In other words, let theoretical mathematicians devise what new theories they please, the practical electrician must work upon the old theory that the conductor does his work and the insulation prevents its being wasted. Ohm's law (based on the old theory) is still his safe guide.

"For this reason I would urge all practical electricians, and all students who desire to gain a clear conception of the actual operations of electricity, to dismiss from their minds the new unproved hypotheses about the ether and the abstract theory of conduction, and to completely master the old, the practical, and common sense theory which links matter and energy together, . . . "

Sprague accurately described Theory N.

One of the few supporters of Theory H was J. A. Fleming, who wrote[5]:

"It is important that the student should bear in mind that, although we are accustomed to speak of the current as *flowing in the wire* in one direction or the other, this is a mere form of words. What we call *the current* in the wire is, to a very large extent, a process going on in the space or material outside the wire. Just as we familiarly speak of the sun rising and setting, when the effect is really due to the rotation of the earth, so the ordinary language we use in speaking about electric currents flowing in conductors retains the form impressed upon it by older and erroneous assumptions as to their nature."

Heaviside's view

As time went by, support for Theory H gradually died out. Let us end Theory H with a long discussion by its originator[6]:

"Consider the electric current, how it flows. From London to Manchester, Edinburgh, Glasgow, and hundreds of other places, day and night, are sent with great velocity, in rapid succession, backwards and forwards, electric currents, to effect mechanical motions at a distance, and thus serve the material interests of man.

* Phlogiston was a 'subtle fluid' postulated by the German chemist G. E. Stahl (1660-1734). It was thought to be combined with a 'calx' or ash in combustible materials and to be given off by these materials in the process of burning, leaving the ash behind. This hypothesis was strongly held in the 18th century but was eventually upset by Lavoisier's deductions leading to the theory of the conservation of mass. – Ed.

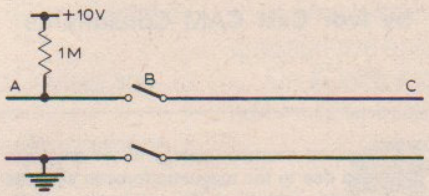
"By the way, is there such a thing as an electric current? Not that it is intended to cast any doubt upon the existence of a phenomenon so called; but is it a current – that is, something moving through a wire? Now, although nothing but very careful inculation at a tender age, continued unremittingly up to maturity, of the doctrine of the materiality of electricity, and its motion from place to place, would have made me believe it, still, there is so much in electric phenomena to support the idea of electricity being a distinct entity, and the force of habit is so great, that it is not easy to get rid of the idea when once it has been formed. In the historical development of science, static phenomena came first. In them the apparent individuality of electricity, in the form of charges upon conductors, is most distinctly indicated. The fluids may be childish notions, appropriate to the infancy of science; but still electric charges are easily imaginable to be quantities of a something, though not matter, which can be carried about from place to place. In the most natural manner possible, when dynamic electricity came under investigation, the static ideas were transferred to the electric current, which became the actual motion of electricity through a wire. This has reached its fullest development in the hands of the German philosophers, from Weber to Clausius, resulting in ingenious explanations of electric phenomena based upon forces acting at a distance between moving or fixed individual elements of electricity.

"Return to our wire from London to Edinburgh with a steady current from the battery in London. The energy is poured out of the battery *sideways* into the dielectric at a steady rate. Divide into tubes bounded by lines of energy-current. They pursue in general solenoidal paths in the dielectric, and terminate in the conductor. The amount of energy entering a given length of the conductor is the same wherever that length may be situated. The lines of energy-current are the intersections of the magnetic and electric equipotential surfaces. Most of the energy is transmitted parallel to the wire nearly, with a slight slant towards the wire in the direction of propagation; thus the lines of energy-current meet the wire very obliquely. But some of the outer tubes go out into space to an immense distance, especially those which terminate on the further end of the wire. Others pass between the wire and the earth, but none in the earth itself from London to Edinburgh, or vice versa, although there is a small amount of energy entering the earth straight downwards, especially at the earth "plates". If there is an instrument in circuit at Edinburgh, it is worked by energy that has travelled wholly through the dielectric, then finding its way into the instrument . . ."

If we keep to Theory H, the theory that the field $E \times H$, travelling along between the wires at the speed of light – what Heaviside called the 'energy current', is the cause, then electric charge and electric current are merely what define the *edge* of an energy current. If electric current is that which defines the side of an energy current, then we may with equal justification postulate 'displacement current' as that which defines the front face of a step of energy current[1].

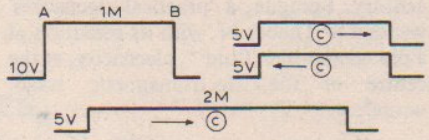
Now let us move on to Theory C, when we drop the dualism – circuit and field – that has until now been the foundation of electromagnetic theory. First we shall discuss the reed relay pulse generator, which illustrates some of the ideas underlying Theory C.

The reed relay pulse generator was a means of generating a fast pulse using rather primitive methods. A one-metre section of 50-ohm coaxial cable AB was charged up to a steady 10 volts (say) via a one megohm resistor, and then suddenly discharged into a long piece of coax BC by the closure of two switches.



A five-volt pulse two metres wide was found to travel off to the right at the speed of light for the dielectric on closure of the switches, leaving the section AB completely discharged. (The practical device lacked the second, lower switch at B, which is added in the diagram to simplify the argument).

The curious point is that the width of the pulse travelling off down BC is twice as much as the time delay for a signal between A and B. Also, the voltage is half of what one would expect. It appears that after the switch was closed, some energy current must have started off to the *left, away* from the now closed switch; bounced off the open circuit at A, and then returned all the way back to the switch at B and beyond.



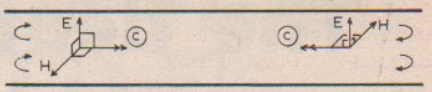
This paradox, that when the switches are closed, energy current promptly rushes *away* from the path suddenly made available, is understandable if one postulates that a steady charged capacitor is not steady at all; it contains energy current, half of it travelling to the right at the speed of light, and the other half travelling to the left at the speed of light.

Now it becomes obvious that when the switches are closed, the right-wards travelling energy current will exit down BC first, immediately followed by the leftwards travelling energy current after it has bounced off the open circuit at A.

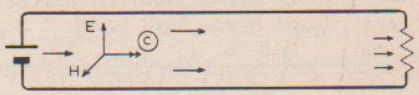
We are driving towards the principle that *energy (current) $E \times H$ cannot stand still; it can only travel at the speed of light.* Any apparently steady field is a combination of two energy currents travelling in opposite directions at the speed of light[7].

E and H always travel together in fixed proportion Z_0.

Electric charge does not exist according to Theory C. The so-called electric charge is merely the edge of two reciprocating energy currents. In the case of the so-called steady charged capacitor, the electric fields of the two energy currents add but the magnetic fields cancel, so that it has come to be thought that a charged capacitor is devoid of magnetic field.



Now let us consider a simple circuit with battery and resistor. Two conductors guide the energy current from battery to resistor. It enters the resistor *sideways*



(Kip 1962)[6]. 'Electric current' is merely the side of a wave of energy current. If a 'conductor' is perfect, the energy current has a sharp side; the so-called 'electric current' has infinite density in the outside surface of the 'electric conductor', which Heaviside called an obstructor.

Energy current penetrates an imperfect conductor in the same way as it enters a resistor, from the side. In this case, the region containing a variation in energy current density, the so-called 'electric current', widens and penetrates into the conductor; skin depth is no longer zero.

Nothing exists behind a mirror; nothing happens there. The velocity of the 'things' behind a mirror does not depend on the medium, or material, behind the mirror[8].

As Maxwell's equations show,[9] 'electric current' is always derivable as the gradient on the side of a wave of energy current. Unlike energy current (but like the images in a mirror), electric current contains no energy, it has no function, and it explains nothing. Electric current does not exist.

Although a cloud cannot exist without edges, the *edges* of a cloud do not exist. They have no width, volume, or materiality. However, the *edges* of a cloud can be drawn. Their shapes can be manipulated graphically and mathematically. The same is true of the so-called 'electric current'.

In the following analogies, the sheep represent energy, the dogs electricity.

Theory N. The sheep are forced out of the pen by the sheep-dogs. The dogs then run alongside the sheep. There can only be a forward flow if sheep-dogs first advance on both sides of the flow of sheep, which the dogs direct and cause.

Theory H. The sheep rush out of the pen into the great open spaces. They will go forward regardless, but their direction is actively guided by the sheep-dogs running alongside, the front of the line of dogs always keeping level with the foremost sheep.

Theory C. There are no sheep-dogs. The sheep leave the pen and flow out into the great open spaces. Some of the space is rougher. (This rough space was previously thought to be the terrain preferred by the dogs.) Here fewer sheep go, and their rate of advance is slower. Some ground is very obstructive, nearly impassable for sheep.

Although it might appear that the sheep are actively guided by the rough terrain towards the smooth terrain, this is not so. Neither does a grease mark on blotting paper actively guide the ink towards the ungreasy areas. There is no active guidance mechanism; greasy paper is merely bad blotting paper with poor capillary action, passively guiding the ink.

The excision of sheep-dogs from the theory is a giant simplification. Nothing flows in the conductor; nothing happens therein. Heaviside was right to call it an obstructor. Half of the primitives in electromagnetic theory disappear, and it ceases to be a dualistic theory. ρ and $\mathcal{J}$ disappear, becoming merely the physically non-existent results of the mathematical manipulation of E and H, with no more significance than "circularity"' (Letters, June 1979 issue, p. 82).

The direct transition from Theory N to Theory C is similar to the change in combustion theory from phlogiston to oxidation, but is more difficult. Phlogiston is very similar to electricity, being a strange 'fluid' which permeates solids. But whereas the oxygen which 'replaced' phlogiston was still within the same body, the energy current which replaces electricity is not where the electricity was; it is where it was not. This is a very difficult transition. If the idea of replacing the well known phlogiston by oxygen caused mirth at High Table, we have to expect Theory C to generate widespread hilarity.

I would like to thank David Walton and Malcolm Davidson of CAM Consultants for their dogged support for six years. This article is taken from the book Electromagnetic Theory Vol 2, pub. CAM Publishing, 17 King Harry Lane, St. Albans, England.

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Appendix

Definition of a perfect conductor: $\epsilon=\infty$.
It follows that velocity of energy current

$$=\frac{1}{\sqrt{\mu\epsilon}}=0$$

Impedance $Z_0=\sqrt{(\mu/\epsilon)}=0$

In an imperfect conductor, ϵ is very high.
Impedance $(=Z_0)\rightarrow 0$
Penetration velocity is very slow. □