The capacitor

In the early 1960's I pioneered the inter-connection of high speed (1nsec) logic gates at Motorola, Phoenix, Arizona ([ref.25](#)). One of the problems to be solved was the nature of the voltage decoupling at a point given by two parallel voltage planes. I asked Bill Herndon about the problem, and he gave me the answer: "It's a transmission line". Bill learnt this from Stopper, whom I never met, who later worked for Burroughs (UNISYS) Corp. in Detroit.

The fact that parallel voltage planes, when entered at a point, present a resistive, not a reactive, impedance, was for me an important breakthrough. (It meant that as logic speeds increased, there would be no limitation presented by the problem of supplying +5v.) The reader should be able to grasp the reason why voltage plane decoupling is resistive by studying Figure 64, which shows the effect of a segment only of two planes as they are seen from a point.

During the next ten years, with the help of Dr. D. S. Walton, I gradually came to appreciate that, since a conventional capacitor was made up of two parallel voltage planes it also had a resistive, not a reactive (i.e. capacitive or inductive) source impedance when used to decouple the +5v supply to logic. Since the source impedance (= transmission line characteristic impedance) is well below one ohm, the transient current demand of logic gates approaching infinite speed can still be successfully satisfied with +5v from a standard capacitor of any type([2](#))

The capacitor is an energy store, and when energy is injected, it enters the capacitor sideways at the point where the two leads are joined to the capacitor. Nothing ever traverses a capacitor from one plate to the other([3](#)). This is clearly understood in the case of a transmission line. By definition, when a TEM wave travels down a transmission line, Figure 5, nothing travels sideways across the transmission line, or we would not have a transverse electromagnetic wave.

Comparison of the transmission line model with the lumped model of a capacitor in an RC circuit.

Consider a transmission line as shown in Figure 65 with characteristic impedance $Z_0$ terminating in an open circuit. We will assume that $R \gg Z_0$.

When the switches are closed (at time $t=0$) a step of voltage $V \frac{Z_0}{R + Z_0}$ is propagated down the line. This reflects from the open circuit at the right hand end to give a total voltage of $2V \frac{Z_0}{R + Z_0}$. Reflection from the left hand end makes a further contribution of $V \frac{Z_0}{R + Z_0} \frac{R - Z_0}{R + Z_0}$ and so on. In general, after $n$ two-way

passes the voltage is $V_n$ and;

$$V_{n+1} = V_n + 2V \frac{Z_0}{R + Z_0} \left[ \frac{R - Z_0}{R + Z_0} \right]^n$$


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[2](#) The reader should be able to grasp the reason why voltage plane decoupling is resistive by studying Figure 64, which shows the effect of a segment only of two planes as they are seen from a point.

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[4](#) Comparison of the transmission line model with the lumped model of a capacitor in an RC circuit.

[5](#) Consider a transmission line as shown in Figure 65 with characteristic impedance $Z_0$ terminating in an open circuit. We will assume that $R \gg Z_0$.

[6](#) When the switches are closed (at time $t=0$) a step of voltage $V \frac{Z_0}{R + Z_0}$ is propagated down the line. This reflects from the open circuit at the right hand end to give a total voltage of $2V \frac{Z_0}{R + Z_0}$. Reflection from the left hand end makes a further contribution of $V \frac{Z_0}{R + Z_0} \frac{R - Z_0}{R + Z_0}$ and so on. In general, after $n$ two-way

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In order to avoid a rather difficult integration it is possible to sum the series to \( n \) terms using the formula

\[
\sum_{n=1}^{\infty} V = a \frac{1-v^n}{1-v} \quad (2)
\]

where \( a \) is the first term of a geometrical progression and \( v \) the ratio between terms. (This formula is easily verified by induction.) Substituting in (2) the parameters for (1),

\[
a = 2V \frac{Z_0}{R + Z_0} \quad (3)
\]

\[
V = \frac{R - Z_0}{R + Z_0} \quad (4)
\]

We obtain,

\[
V_n = 2V \left( \frac{Z_0}{R + Z_0} \right) \left[ 1 - \left( \frac{R - Z_0}{R + Z_0} \right)^n \right] \quad (5)
\]

\[
= V \left[ 1 - \left( \frac{R - Z_0}{R + Z_0} \right)^n \right] \quad (6)
\]

This is a correct description of what is happening as a capacitor charges. We can now go on to show that it is approximated by an exponential. We have

\[
V_n = V \left[ 1 - \left( \frac{R - Z_0}{R + Z_0} \right)^n \right] \quad (7)
\]

Consider the term,

\[
T = \left( \frac{R - Z_0}{R + Z_0} \right)^n
\]
If \( Z_0/R \ll 1 \) this term is asymptotically equal to

\[
T = \left( 1 - \frac{2Z_0}{R} \right)^n.
\]

Now define

\[ k = 2Z_0 \frac{n}{R}. \]

Substitution gives

\[ T = \left( 1 - \frac{k}{n} \right)^n. \]

By definition, as \( n \to \infty \), we have,

\[ T = e^{-k} = e^{-\frac{2Z_0 n}{R}}. \]

and therefore:

\[ V_n = V \left( 1 - e^{-\frac{2Z_0 n}{R}} \right). \]

Now, after time \( t \), \( n = \frac{C}{2l} \), where \( C \) = velocity of propagation.

Thus,

\[ V(t) = V \left[ 1 - e^{-\frac{Ct}{1 \frac{Z_0}{R}}} \right]. \]

For any transmission line it can be shown (p19) that

\[ Z_0 = \sqrt{\frac{\mu}{\varepsilon}}, \quad C = \frac{1}{\sqrt{\mu \varepsilon}}, \quad c = \frac{\varepsilon}{f} \]
where $c =$ capacitance per unit length and $f$ is a geometrical factor in each case. The "total capacitance" of length $l$

$$= lc = C.$$ 

Hence,

$$\frac{CZ_0}{IR} = \frac{1}{RC}$$

and therefore

$$V(t) = V\left[1 - e^{-\frac{t}{RC}}\right]$$

which is the standard result. This model does not require use of the concept of charge. A graphical comparison of the results is shown in Figure 66. 

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Ref.15, p40.

Ref.3b, p216, refutes the fashionable nonsense about "RF capacitors".

Similarly, the battery, p13, note 24, and the electrolyte.

Calculations were by my co-author Dr. D.S. Walton. First published in Wireless World, dec78, p51.