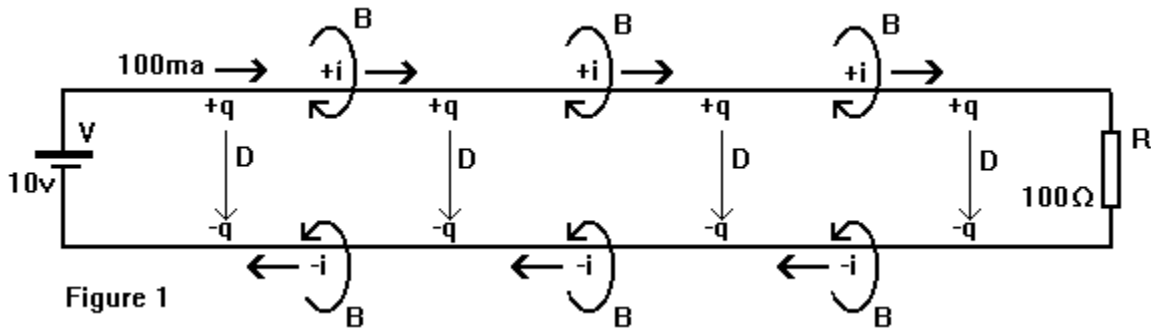


Battery and resistor. Steady state. Numerical values.

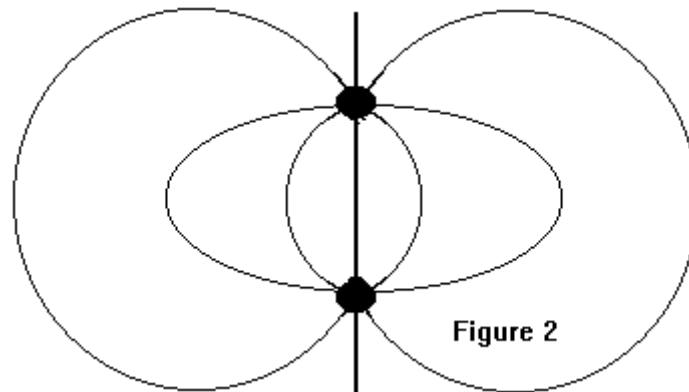
We start with a conventional view of a 10 volt battery connected via two uniform conductors to a 100 Ω resistor (Fig.1).



A steady current flows round the circuit, through battery, conductors and resistors. Ohm's Law tells us that the voltage equals the current multiplied by the resistance. Therefore the current is

$$\frac{10}{100} \text{ amp} = 100 \text{ ma}$$

Every point on the surface of the upper conductor is at a potential V of 10v, and every point on the surface of the lower conductor is at a potential of 0v.



The space between the two conductors, shown in cross section (Fig.2), is filled by tubes of electric displacement D. Each tube of electric displacement terminates on unit positive charge on the upper conductor and unit negative charge on the lower conductor. If the capacitance between the two conductors is C, then the total charge on each conductor is given by Q = CV. If the capacitance per foot is c, then the charge per foot is q = cV.

The energy stored in the electric field between the conductors is

$$\frac{1}{2} CV^2 \text{ (Ref. 1), or } \frac{1}{2} cV^2 \text{ per unit length.}$$

The space between the two conductors is filled by tubes of magnetic flux which encircle the current in the conductor.

If the self inductance of the pair of conductors is L, then the total magnetic flux passing between the conductors

is $\Phi = L I$. If the inductance per unit length is l , then the magnetic flux per unit length is $\phi = l I$.

The energy stored in the magnetic field created by the current in the two conductors is

$$\frac{1}{2} L I^2 \text{ (Ref. 2), or } \frac{1}{2} l I^2 \text{ per unit length.}$$

Power is delivered by the battery into the circuit at the rate of watts which is the product of voltage and current. This equals $10\text{v} \times 100\text{ma} = 1 \text{ watt}$. The resistor absorbs energy at the same rate, and turns 1 watt of electric power into heat, which then radiates from it.

The energy trapped in the fields between the conductors totals

$$\frac{1}{2} C V^2 + \frac{1}{2} L I^2 \quad (1)$$

The energy stored in each unit length is

$$\frac{1}{2} c V^2 + \frac{1}{2} l I^2 \quad (2)$$

Battery and resistor. Initial state.

Now let us turn to the conventional view of the initial conditions. We will insert two switches, one in the top conductor and one in the bottom conductor (Fig. 3).

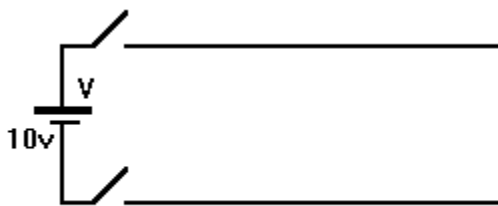


Figure 3

When we close the two switches, the distant resistor cannot define the current which rushes along the wires because the wave front has not yet reached the resistor (Figs.4,5.)

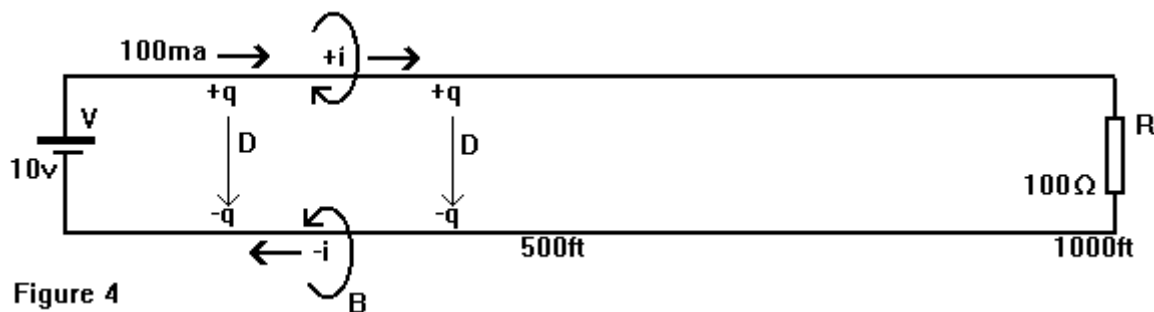


Figure 4



Figure 5

Lacking knowledge of the value of the resistor, the current is defined by the characteristic resistance Z_0 of the pair of conductors (usually called their characteristic impedance. Thus, $\frac{V}{I} = Z_0$.)

In the case of the cross section shown (Fig.2), let us assume this is about $100 \Omega^2$. So the instantaneous current is 100ma. Instead of delivering the 1w (=1J/s) of power to the resistor, the battery delivers it into the space between the conductors for the first few nanoseconds. The wave front travels to the right at the speed of light for the vacuum, that is, one foot per nanosecond [2]. In our case, where the resistor is 1000 feet from the battery, the wave front reaches the resistor after $1 \mu\text{sec}$. During this initial $1 \mu\text{sec}$, the battery supplies the energy necessary (eqn.1) to set up the electric and magnetic fields in the space between the conductors. The energy delivered by the battery during the $1 \mu\text{sec}$, when the wave front travels from battery to resistor is VIt , where t is $1 \mu\text{sec}$. This equals $1 \mu\text{J}$, or 1nJ per foot.

The energy per foot in the electric field is $\frac{1}{2} cV^2$, where c is the capacitance per foot between the conductors.

For a 100Ω line, this is 10 pF^3 , resulting in energy of about $\frac{1}{2} \text{ nJ}$. The energy per foot in the magnetic field is

$\frac{1}{2} l I^2$, where l is the self inductance of the loop formed by one foot length of the two conductors. The

inductance is about 100nH [3], resulting in energy of about $\frac{1}{2} \text{ nJ}$.

The characteristic resistance is

$$Z_0 = \sqrt{\frac{l}{c}} . \text{ (Ref. 3) } (3)$$

The above calculations showed that in the initial (transient) case, electric and magnetic energy are equal. Simple algebra will give the same result, as follows;

The energy in the electric field is

$$u_d = \frac{1}{2} cV^2 .$$

Using the formula

$$\frac{V}{I} = Z_0, \text{ which means that } V^2 = I^2 Z_0^2,$$

and we can rewrite

$$\frac{1}{2} c V^2 \text{ as } \frac{1}{2} c I^2 Z_0^2.$$

Now substitute

$$\frac{l}{c} \text{ for } Z_0^2$$

and we get

$$\frac{1}{2} l I^2$$

the energy in the magnetic field. Each of these equals $\frac{1}{2}$ nJ per foot.

Power from the battery is VI. One second's worth of this power would charge up 10^9 feet of cable, because the velocity of propagation C is 10^9 feet per second. So the energy stored in one foot length is $U = VI/C = 1\text{nJ}$. So the energy delivered by the battery in 1 nsec equals the energy stored in the fields in a section one foot long.

If the terminating resistor is equal to the transmission line's characteristic impedance, then there is no reflection. The battery thinks the transmission line has infinite length. It continues to deliver power at the initial rate of 1 watt.

Unterminated transmission line.

If the resistor is missing, then all of the energy travelling to the right at the speed of light is reflected and begins the return journey to the left (Fig.6).



Figure 6

Let us consider the case where the line length is 1000 feet, and 1500 nsec have elapsed since the switches were closed. The field situation in the first 500 feet is as before, the energy being $\frac{1}{2}$ nJ per foot in the electric field and $\frac{1}{2}$ nJ per foot in the magnetic field. In the last 500 feet, a returning wave front of equal energy density is superposed on the energy making its outward journey. Magnetic fields cancel out, and we appear to have a

steady charged capacitor 500 feet long, charged to an amplitude of 20 volts. Our formula

$$U = \frac{1}{2} CV^2$$

gives energy per unit length as before. Since the voltage has doubled, the energy appears to be 2nJ instead of the $\frac{1}{2}$ nJ associated previously with the electric field. Thus, double the electric field has led to four times the energy. This is untrue, because the two electric fields, one travelling to the right and the other to the left, have no relationship with each other. The reality is that each electric field contains $\frac{1}{2}$ nJ per foot, totalling 1nJ per foot. The missing energy is contained in the invisible magnetic fields, invisible because the leftwards travelling magnetic field makes the equal rightwards travelling magnetic field invisible to our measuring instruments. In the last 500 feet, the energy per foot is made up as follows;

$$\frac{1}{2} \text{ nJ in the forward travelling electric field,}$$

$$\frac{1}{2} \text{ nJ in the forward travelling magnetic field,}$$

$$\frac{1}{2} \text{ nJ in the backward travelling electric field, and}$$

$$\frac{1}{2} \text{ nJ in the backward travelling magnetic field.}$$

It is a mathematical accident that we get the correct answer for total energy when we wrongly think that the last 500 feet are steadily charged with

electric field, and no magnetic field exists. Pace our calculations, the total energy density from electric fields is 1nJ not 2nJ.

^[1] This is Gauss's Law.

^[2] See Ref.3a, pp 5 and 8 (eqn.3.5);

$C=1/(\tilde{A}-m_0e_0)=1/(\tilde{A}-lc)$. Also p.95.

^[3] *ibid*, 2 $l=0.4 \ln(a/r)$ mH/m, giving us about 100nH/ft.

(I write 2 l not l because there are two conductors.)

